Astroparticle Physics, NAASPH-12.2013-2014.2A

Ad van den Berg & Olaf Scholten
KVI - Center for Advanced Radiation Technology NL-9747 AA Groningen

Exam; April 28, 2014; 18:30-21:30; A. Jacobshal 01
5 problems (total of 47 points).
Write the solution of every problem on a separate piece of paper with name and student number.

Write clearly, it should be readable.

Problem 1 (10 pnts in total)
Models for acceleration of charged particles in the cosmos can be based on so-called magnetic bottles.

2 pnts a. What are magnetic bottles? Provide a schematic picture of a magnetic bottle.

3 pnts b. Describe the role of magnetic bottles in the acceleration process of charged particles.

5 pnts c. Explain that the direction of the force acting on a charged particle in a magnetic bottle is proportional to $-\frac{\partial B_z}{\partial z}$, where z is the symmetry axis of the bottle.
Hint: assume that the magnetic field strength in a magnetic bottle has cylindrical symmetry with only components in the z and r direction and use that the divergence operator in cylindrical coordinates (r, ϕ, z) can be written as:

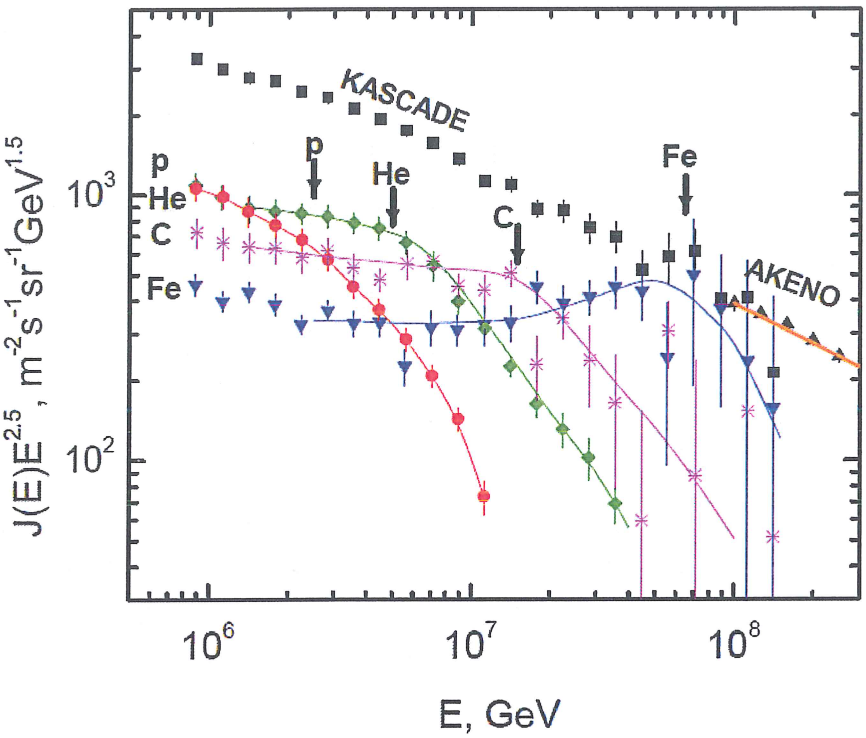
$$\vec{\nabla} \cdot \vec{B} = \frac{1}{r}\frac{\partial(rB_r)}{\partial r} + \frac{1}{r}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}$$

Problem 2 (8 pnts in total)

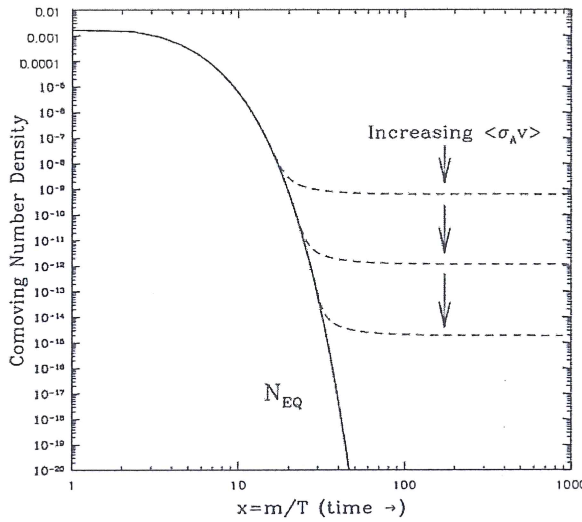
2 pnts a. Which astronomical objects are being considered to act as the sources of the **Galactic** cosmic rays.

3 pnts b. Give (at least) two reasons why scientists have arrived at this conclusion.

3 pnts c. Give a possible explanation of the cosmic-ray flux spectrum up to an energy of 10^{17} eV; see the figure.



Problem 3 (8 pnts in total)
Dark Matter has not yet been proven to exist. However, there are many indications that the energy density in the Universe today should be attributed for more than 20% to Dark Matter (DM). The figure shows the density of DM in the universe as function of time or temperature.



4 pnts a. Name at least 4 observations from which evidence arises for Dark Matter and describe how they test the presence of DM.

2 pnts b. As shown in the figure the density of DM particles may be high during the early stage of the Big Bang, but becomes constant at a certain stage. Explain why this happens and why the final density decreases with increasing $< \sigma v >$ as indicated.

2 pnts c. Explain why left-handed neutrinos (the normal weak-interacting ones) do not form a large fraction of DM (give two reasons).

Problem 4 (12 pnts in total)

When high-energy cosmic rays enter the atmosphere of the Earth they create a cascade of secondary particles, called an extensive air shower. Such a shower generally has a hadronic (protons, neutrons, pions) and an electromagnetic component (photons, electrons, positrons). In addition the shower contains an increasing number of neutrinos and muons.

2 pnts a. Indicate the reaction processes in the extensive air shower induced by a cosmic-ray proton that create the electromagnetic component.

2 pnts b. Same question for the neutrino and muon content of the shower.

2 pnts c. Indicate the general structure of the shower profile, the number of charged particles as function of height and give the reason why this differs from that for the neutrino and muon components.

2 pnts d. Which component of the shower can be measured with scintillator detectors at the surface of the Earth and give the reasons.
Indicate qualitatively how the strength of the signal depends on the distance to the core of the shower.

2 pnts e. What process induces air-fluorescence emission from an extensive air shower?
Which aspects of the shower can be measured by this process and describe how this measurement is performed in practice.

2 pnts f. Discuss the observables that can distinguish if cosmic rays at the highest energies are: all protons, all iron, or a mixture of the two.

Problem 5 (9 pnts in total)

A high energy electron in a shower scatters off a nucleon (mass M, at rest) in an air-molecule. Assume that the electron, moving in the $\hat{z}$-direction, has an energy of $E_e = 10^{16}$ eV and scatters over a small angle θ of order mili-radians. In the scattering process it has lost half its energy and thus $E'_e = 5 \times 10^{15}$ eV. The electron has transferred energy and momentum (four momentum q^μ) to one of the quarks inside the nucleon which has a momentum fraction x_{bj}.

3 pnts a. Calculate the components of q^μ. Show that for the energies and angles in this problem $q^2 \approx -2\, E_e E'_e(1 - \cos\theta) + \mathcal{O}(m_e^2 E_e/E'_e) \approx -10^{32}\theta^2/2$ eV^2.

1 pnts b. Show that the momentum fraction of the parton is given by $x_{bj} = -q^2/(2M(E_e - E'_e))$.

1 pnts c. Calculate the maximal scattering angle for the electron for the given energy loss.

2 pnts d. Assume that the hit quark has color-charge *red*. What is the color of the remaining di-quark. Indicate in a simple figure where a color-string will be formed.

2 pnts e. Calculate the invariant mass of the color-string that is formed when the electron scattering angle equals $\theta = 10^{-4}$ radian.

Some numbers

Electron mass $m_e c^2 = 511\,\mathrm{keV}$; Muon mass $m_\mu c^2 = 106\,\mathrm{MeV}$; Pion mass $m_\pi c^2 = 140\,\mathrm{MeV}$;
Proton mass: $m_p c^2 = 0.938\,\mathrm{GeV}$
Conversion: $1\,\mathrm{eV/c^2} = 1.78 \times 10^{-36}\,\mathrm{kg}$
Boltzmann's constant: $k = 8.62 \times 10^{-11}\,\mathrm{MeV/K}$
Planck's constant: $h = 4.1 \times 10^{-15}\,\mathrm{eV\ s}$
Avogadro's number: $N_A = 6 \times 10^{23}\,\mathrm{/mol}$
Solar Mass: $\mathrm{M}_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Parsec: $1\,\mathrm{pc} = 3.1 \times 10^{16}\,\mathrm{m}$
Velocity of Sun w.r.t. center Milkyway: $V_\odot = 270\,\mathrm{km/s}$
Velocity of Earth in orbit around the Sun: $V_\oplus = 30\,\mathrm{km/s}$
Typical galactic dark matter density: $\rho_{DM} = 9\ \mathrm{k\ M_\odot/pc^3}$

Air-shower physics

At 10 km height the density of the atmosphere is $0.4 \times 10^{-3}\,\mathrm{g\,cm^{-3}}$.
The penetration depth for pions in air is $\lambda_\pi = 120\,\mathrm{g\,cm^{-2}}$, for protons $\lambda_p = 90\,\mathrm{g\,cm^{-2}}$, and for iron is $\lambda_{Fe} = 5\,\mathrm{g\,cm^{-2}}$.
The mean travel distance in vacuum of a pion with energy E is $d_{\pi^0} = \gamma 25 \times 10^{-9}\,\mathrm{m}$ and $d_{\pi^\pm} = \gamma 7.8\,\mathrm{m}$ where the relativistic γ-factor is given by $\gamma = E/mc^2$ and $m_\pi c^2 = 140\,\mathrm{MeV}$.

Integrals

For $c > 0$ we have:

$$\int_0^\infty c\,e^{-cx}dx = 1\,; \quad \int_0^\infty c\,x\,e^{-cx}dx = 1/c\,; \quad \int_0^\infty c\,x^2 e^{-cx}dx = 2/c^2$$